



Printed Pages : 7

EAS-103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

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B. Tech.

(Only for the candidates admitted/Readmitted in the session 2008-09)

(SEM. I) EXAMINATION, 2008-09

MATHEMATICS - I

Time : 3 Hours]

[Total Marks : 100

SECTION - A

All parts of this question are compulsory.

2×10=20

1 (a) For which value of ' b ' the rank of the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix} \text{ is } 2, b = \underline{\hspace{2cm}}$$

(b) Determine the constants a and b such that

the curl of vector $\vec{A} = (2xy + 3yz)\hat{i} +$

$(x^2 + axz - 4z^2)\hat{j} - (3xy + byz)\hat{k}$ is zero,

$a = \underline{\hspace{2cm}}, b = \underline{\hspace{2cm}}$.

(c) The n^{th} derivative (y_n) of the function

$y = x^2 \sin x$ at $x = 0$ is $\underline{\hspace{2cm}}$.



(d) With usual notations, match the items on right hand side with those on left hand side for properties of Max^m and minimum. :

- | | |
|---------------------|--------------------------------|
| (i) Max^m | (p) $rt - s^2 = 0$ |
| (ii) Min^m | (q) $rt - s^2 < 0$ |
| (iii) Saddle point | (r) $rt - s^2 > 0, r > 0$ |
| (iv) Failure case | (s) $rt - s^2 > 0$ and $r < 0$ |

(e) Match the items on the right hand side with those on left hand side for the following special functions : (Full marks is awarded if all matchings are correct)

- | | |
|---|---|
| (i) $\beta(p, q)$ | (p) $\sqrt{(1/2)}$ |
| (ii) $\frac{\sqrt{p} \sqrt{q}}{\sqrt{p+q}}$ | (q) $\int_0^\infty \frac{y^{p-1}}{(1+y)(p+q)} dy$ |
| (iii) $\sqrt{\pi}$ | (r) $\beta(p, q)$ |
| (iv) $\frac{\pi}{\sin p \pi}$ | (s) $\sqrt{p} \sqrt{1-p}$ |

Indicate True or False for the following statements :

- (f) (i) If $|A| = 0$, then at least one eigen value is zero. (True / False)
- (ii) A^{-1} exists iff 0 is an eigen value of A . (True / False)
- (iii) If $|A| \neq 0$, then A is known as singular matrix. (True / False)

(iv) Two vectors X and Y is said to be orthogonal $Y, X^T Y = Y^T X \neq 0$. (True / False)

- (g) (i) The curve $y^2 = 4ax$ is symmetric about x -axis. (True / False)
- (ii) The curve $x^3 + y^3 = 3axy$ is symmetric about the line $y = -x$. (True / False)
- (iii) The curve $x^2 + y^2 = a^2$ is symmetric about both the axis x and y . (True / False)
- (iv) The curve $x^3 - y^3 = 3axy$ is symmetric about the line $y = x$. (True / False)

Pick the correct answer of the choices given below :

(h) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is position vector, then value of $\nabla(\log r)$ is

- | | |
|------------------------------|----------------------------|
| (i) $\frac{\vec{r}}{r}$ | (ii) $\frac{\vec{r}}{r^2}$ |
| (iii) $-\frac{\vec{r}}{r^3}$ | (iv) none of the above. |

(i) The Jacobian $\frac{\partial(uv)}{\partial(xy)}$ for the function

$u = e^x \sin y, v = (x + \log \sin y)$ is

- | | |
|---------|---|
| (i) 1 | (ii) $\sin x \sin y - xy \cos x \cos y$ |
| (iii) 0 | (iv) $\frac{e^x}{x}$ |



- (j) The volume of the solid under the surface $az = x^2 + y^2$ and whose base R is the circle $x^2 + y^2 = a^2$ is given as

- (i) $\pi |2a|$ (ii) $\pi a^3 |2|$
 (iii) $\frac{4}{3}\pi a^3$ (iv) None of the above.

SECTION - B

Attempt any **three** parts of the following :

10×3=30

- 2 (a) If $y = (\sin^{-1} x)^2$ prove that $y_n(0) = 0$ for b odd and $y_n(0) = 2, 2^2, 4^2, 6^2 \dots (n-2)^2$, $n \neq 2$ for n is even.
 (b) Find the dimension of rectangular box of maximum capacity whose surface area is given when (a) box is open at the top (b) box is closed.
 (c) Find a matrix P which diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, verify $P^{-1}AP = D$ where D is the diagonal matrix.
 (d) Find the area and the mass contained m the first quadrant enclosed by the curve $\left(\frac{x}{a}\right)^\alpha + \left(\frac{y}{b}\right)^\beta = 1$ where $\alpha > 0$, $\beta > 0$ given that density at any point $p(xy)$ is $k\sqrt{xy}$.

- (e) Using the divergence theorem, evaluate the surface integral $\iint_S (yz \, dy \, dz + zx \, dz \, dx + xy \, dy \, dx)$ where $S : x^2 + y^2 + z^2 = 4$.

SECTION - C

Attempt any **two** parts from each question. All questions are compulsory.

- (a) Trace the curve $r^2 = a^2 \cos 2\theta$
 (b) If $u = \log \left(\frac{(x^2 + y^2)}{(x + y)} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$
 (c) If $V = f(2x - 3y, 3y - 4z, 4z - 2x)$, compute the value of $6V_x + 4V_y + 3V_z$.
- 4 (a) The temperature ' T ' at any point (xyz) in space is $T(xyz) = Kxyz^2$ where K is constant. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$.
 (b) Verify the chain rule for Jacobians if $x = u$, $y = u \tan v$, $z = w$.



- (c) The time 'T' of a complete oscillation of a simple pendulum of length 'L' is governed by the equation

$$T = 2\pi \sqrt{\frac{L}{g}}, \quad g \text{ is constant, find the approximate}$$

error in the calculated value of T corresponding to an error of 2% in the value of L .

- 5 (a) Determine 'b' such that the system of homogeneous equation $2x + y + 2z = 0$; $x + y + 3z = 0$; $4x + 3y + bz = 0$ has (i) Trivial solution (ii) Non-Trivial solution. Find the Non-Trivial solution using matrix method.

- (b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \text{ and hence find } A^{-1}.$$

- (c) Find the eigen value and corresponding eigen vectors of the matrix

$$I = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

- 6 (a) Find the directional derivative of $\nabla(\nabla f)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$ where $f = 2x^3y^2z^4$.

- (b) Using Green's theorem, find the area of the region in the first quadrant bounded by the curves

$$y = x, \quad y = \frac{1}{x}, \quad y = \frac{x}{4}.$$

- (c) Prove that $(y^2 - z^2 + 3yz)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

- 7 (a) Changing the order of integration of

$$\int_0^\infty \int_0^\infty e^{-xy} \sin nx \, dx \, dy$$

$$\text{Show that } \int_0^\infty \left(\frac{\sin nx}{x} \right) dx = \frac{\pi}{2}.$$

- (b) Determine the area bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$.
- (c) For a β function, show that $\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$.

